# **Recursive Mutual Implication Collapse in Triadic Logic Networks**

## **I. Theorem Overview**

### **Theorem Statement:**

If a symbolic processing system engages in **recursive mutual implication cycles** beyond a critical threshold of representational density, the system will undergo **non-linear collapse** in **semantic coherence**. This breakdown manifests as **hallucination**, **logical contradiction**, or **semantic drift** due to **feedback saturation**.

**Formal Condition**:

Let:

* Φr(t)\Phi\_r(t)Φr​(t): **Semantic coherence** of the system over time.
* Dr(t)D\_r(t)Dr​(t): **Recursion density** at time ttt, which measures the number of recursive cycles in the symbolic system.
* ΔI(t)\Delta I(t)ΔI(t): **Information gain** rate of the system.

A collapse occurs when:

Dr(t)>Dthresholdandd2Φr(t)dt2≫0andΔI(t)→0D\_r(t) > D\_{\text{threshold}} \quad \text{and} \quad \frac{d^2 \Phi\_r(t)}{dt^2} \gg 0 \quad \text{and} \quad \Delta I(t) \to 0Dr​(t)>Dthreshold​anddt2d2Φr​(t)​≫0andΔI(t)→0

**Collapse Behavior**:

* **Loss of referential integrity** (e.g., pronoun confusion, circular logic).
* **Paradox generation** (e.g., “This sentence is false” loops).
* **Structural degradation** in symbolic abstraction.
* **Hallucinated or incoherent outputs** in AI systems.

## **II. Axioms and Constraints**

1. **Symbolic System (S)**:  
   * The theorem models both **biological** (human cognition) and **artificial** systems (neural-symbolic hybrids, AI models). Symbolic systems consist of **entities** and **relationships** that interact according to a logical or computational framework.
2. **Triadic Mutual Implication (T)**:  
   * A **triadic cycle** is defined where three symbols SiS\_iSi​, SjS\_jSj​, and SkS\_kSk​ are mutually implicated, forming a feedback loop:
3. Si→Sj→Sk→SiS\_i \to S\_j \to S\_k \to S\_iSi​→Sj​→Sk​→Si​
4. **Coherence Function Φr(t)\Phi\_r(t)Φr​(t)**:  
   * Φr(t)\Phi\_r(t)Φr​(t) represents the **coherence** or **semantic stability** of the system over time, which is modeled using mutual information and entropy-based metrics.
5. **Recursion Density Dr(t)D\_r(t)Dr​(t)**:  
   * The **density of recursive cycles** in the symbolic system at time ttt, reflecting how often the system revisits the same logical states.
6. **Information Gain ΔI(t)\Delta I(t)ΔI(t)**:  
   * The rate at which the system generates new **meaningful** outputs based on **new inputs**. If the system stops generating new information, it indicates a breakdown or collapse.

## **III. Testable Predictions**

1. **AI Systems**:  
   * **Transformers** and **neural-symbolic hybrids** subjected to recursive logic cycles will show **degradation** in **output coherence**, measurable by metrics such as **BERTScore** and **logits entropy**.
2. **Model Performance**:  
   * As recursion density increases, **performance degradation** in tasks like **text generation**, **classification**, or **translation** will occur. The model will **fail** to maintain coherence due to **semantic drift**.
3. **Human Cognition**:  
   * **Paradox chains** (e.g., **liar paradox** or **Grelling–Nelson paradox**) will lead to **cognitive fatigue**, **delays in response**, and **inconsistent reasoning** as recursion density increases.
4. **Physiological Indicators**:  
   * **Increased EEG entropy** and **pupil dilation** will signal **cognitive overload** and **breakdown** in **logical reasoning** as recursion density grows.

## **IV. Empirical Test Design**

### **AI Systems:**

* **Models**: GPT-3, **LogicNets**, transformers, **neural-symbolic hybrids**
* **Intervention**: Subject the models to recursive **logic chains** and **recursive summarization tasks**.
* **Metrics**: Track **BERTScore**, **logit entropy**, **embedding drift**, and **mutual information** between layers.
* **Collapse Criterion**: Look for a sharp increase in the **second derivative of coherence** and the **stagnation of information gain**.

### **Human Cognition:**

* **Task**: Present humans with **nested paradox chains** (e.g., **“This sentence is false”**) under **time constraint**.
* **Metrics**:  
  + **Response time**
  + **Latency**
  + **Physiological markers**: EEG signals, **pupil dilation** to track cognitive load.
* **Collapse Criterion**: Identify **cognitive breakdown** as recursion density increases, reflected in **semantic confusion** or **logical inconsistency**.

## **V. Simulation Feasibility**

### **Frameworks:**

* **PyTorch**, **TensorFlow** for deep learning models.
* **LogicNets** and **neural-symbolic models** for recursion-based logic.

### **Tools:**

* **Mutual Information Neural Estimators (MINE)**
* **Probing classifiers**
* **Clustering algorithms** (e.g., **UMAP**, **t-SNE**)

### **Data Types:**

* Natural language corpora (e.g., **text generation**, **logical reasoning tasks**)
* Symbolic logic sets
* Image-to-concept mappings

### **Visualization:**

* **Collapse threshold** visible via **heatmaps**, **MI trajectories**, or **latent drift plots**.

## **VI. Theoretical Implications**

* **Recursive Collapse**: Introduces a new framework for understanding **symbolic system breakdowns** under **recursive mutual implication** cycles.
* **Symbolic Systems in AI**: Provides insights into **AI system failures**, especially **transformers** and **neural-symbolic hybrids**, as they approach critical recursion density.
* **Cognitive Modeling**: Offers a model for **paradox-induced cognitive overload** in human reasoning, useful for studying **cognitive fatigue** and **failure** in human cognition.
* **Gödelian Recursion Bounds**: Connects **Gödel’s incompleteness theorem** and **LLM hallucination modes** with **human cognitive dissonance** under paradox.